

7. Integral Calculus : The Indefinite and Definite Integral

Rule of Integration

- $\int k dx = kx + c$
 - $\int dx = x + c$
 - $\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$
 - $\int x^{-1} dx = \ln x + c, x > 0$
 $\int x^{-1} dx = \ln|x| + c, x \neq 0$
 - $\int e^{kx} dx = \frac{e^{kx}}{k \ln a} + c$
 - $\int e^{kx} dx = \frac{e^{kx}}{k} + c$ where $\ln e = 1$
 - $\int kf(x) dx = k \int f(x) dx$
 - $\int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx$
 - $\int -f(x) dx = -\int f(x) dx$
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Example

- $\int 3 dx = 3x + c$
- $\int x^2 dx = \frac{1}{2+1} x^{2+1} + c = \frac{1}{3} x^3 + c$
- $\int 5x^4 dx = 5 \int x^4 dx$
 $= 5 \left(\frac{1}{5} x^5 + c \right) = x^5 + c$
- $\int (3x^3 - x + 1) dx = 3 \int x^3 dx - \int x dx + \int dx$
 $= 3 \left(\frac{1}{4} x^4 \right) - \frac{1}{2} x^2 + x + c$
 $= \frac{3}{4} x^4 - \frac{1}{2} x^2 + x + c$
- $\int 3x^{-1} dx = 3 \int x^{-1} dx$
 $= 3 \ln|x| + c$
- $\int 2^{5x} dx = \frac{2^{5x}}{5 \ln 2} + c$
- $\int 9e^{-3x} dx = \frac{9e^{-3x}}{-3} + c$
 $= -3e^{-3x} + c$

7.1 Integration by Parts

จาก $\frac{d}{dx}[f(x)g'(x)] = f(x) \cdot g'(x) + g(x) \cdot f'(x)$

Take integral of the derivative given

$$f(x) \cdot g(x) = \int f(x)g'(x)dx + \int g(x)f'(x)dx$$

นั่นคือ

$$\int f(x)g'(x)dx = f(x) \cdot g(x) - \int g(x)f'(x)dx$$

Example จงทำให้เป็นผลสำเร็จ $\int 4x(x+1)^3 dx$

วิธีทำ ให้ $f(x) = 4x$, $g'(x) = (x+1)^3$, $f'(x) = 4$, $g(x) = \int (x+1)^3 dx$

นั่นคือ $g(x) = \int (x+1)^3 dx = \frac{1}{4}(x+1)^4 + c_1$

$$\begin{aligned}\int 4x(x+1)^3 dx &= f(x) \cdot g(x) - \int g(x)f'(x)dx \\ &= 4x \left[\frac{1}{4}(x+1)^4 + c_1 \right] - \int \left[\frac{1}{4}(x+1)^4 + c_1 \right] (4) dx \\ &= x(x+1)^4 + 4c_1 x - \int [(x+1)^4 + 4c_1] dx \\ &= x(x+1)^4 + 4c_1 x - \frac{1}{5}(x+1)^5 - 4c_1 x + c \\ &= x(x+1)^4 - \frac{1}{5}(x+1)^5 + c\end{aligned}$$

Recheck $y(x) = x(x+1)^4 - \frac{1}{5}(x+1)^5 + c$

$$\begin{aligned}y'(x) &= [x \cdot 4(x+1)^3 + (x+1)^4 \cdot 1] - (x+1)^4 \\ &= 4x(x+1)^3\end{aligned}$$

7.2 Integral Calculus : The Definite Integral

ในการทำ Integrate แบบจำกัดขอบเขตใช้สูตร

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$$

Example

$$\begin{aligned} 1. \int_0^4 6x dx &= \int_0^3 6x dx + \int_3^4 6x dx \\ \int_0^4 6x dx &= 3x^2 \Big|_0^4 = 3(4)^2 - 3(0)^2 = 48 \\ \int_0^3 6x dx &= 3x^2 \Big|_0^3 = 3(3)^2 - 3(0)^2 = 27 \\ \int_3^4 6x dx &= 3x^2 \Big|_3^4 = 3(4)^2 - 3(3)^2 = 21 \end{aligned}$$

Checking the answer $48 = 27 + 21$

2. จงทำให้เป็นผลสำเร็จ ให้หาค่าของ $\int_0^2 \frac{3x^2}{(x^3+1)^2} dx$

ให้ $u = x^3 + 1$ แล้ว $du/dx = 3x^2$ and $dx = du/3x^2$ แทนที่

$$\int \frac{3x^2}{(x^3+1)^2} dx = \int 3x^2 u^{-2} \frac{du}{3x^2} = \int u^{-2} du$$

แทนที่ $u = x^3 + 1$ และด้วยข้อจำกัด

$$\begin{aligned} \int_0^2 \frac{3x^2}{(x^3+1)^2} dx &= -(x^3+1)^{-1} \Big|_0^2 = \frac{-1}{2^3+1} - \frac{-1}{0^3+1} = -\frac{1}{9} + 1 \\ &= \frac{8}{9} \end{aligned}$$

3. จงทำให้เป็นผลสำเร็จ

$$\int_1^3 \frac{4x}{(x+1)^3} dx$$