ECS 232 Economic Statistics 1

Exercise 1

(ให้เลือกทำอย่างน้อย 3 ข้อ)

1. An operation consists of two steps, of which the first can be made in n_1 ways. If the first step is made in the i_{th} way, the second step can be made in n_2 ways.
   (a) Use a tree diagram to find a formula for the total number of ways in which the total operation can be made.
   (b) A student can study 0, 1, 2 or 3 hours for a history test on any given day. Use the formula obtained in part (a) to verify that there are 13 ways in which the student can study at most 4 hours for the teat on two consecutive days.

2. (3) With reference to exercise 1.1, suppose that there is a third step, and if the first step is made in the i_{th} way and the second step in the j_{th} way, the third step can be made in n_{3ij} ways.
   (a) Use a tree diagram to verify that the whole operation can be made in

$$\sum_{i=1}^{n_1}\sum_{j=1}^{n_{2i}} n_{3ij}$$

   different ways.

   (b) With reference to part (b) of exercise 1.1, use the formula of part (a) to verify that there are 32 ways in which the student can study at most 4 hours for the test on three consecutive days.

3. (7) Using Stirling's formula[1] to approximate 2n! and n!, show that

$$\frac{\binom{2n}{n}\sqrt{\pi n}}{2^{2n}} \approx 1$$

4. (20) With reference to the generalized definition of binomial coefficients[2] show that

   (a) $\binom{-1}{r} = (-1)^r$

   (b) $\binom{-n}{r} = (-1)^r\binom{n+r-1}{r}$ for n > 0

5. The five finalists in the Miss Universe contest are Miss Argentina. Miss Belgium, Miss U.S.A., Miss Japan and Miss Norway. In how many ways can the judges choose:

   (a) The winner and the first runner-up.

   (b) The winner, the first runner-up, and the second runner-up?

6. How many permutations are there of the letters in the word

   (a) "great"

   (b) "greet"?

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[1] Stirling's formula: When n is large, n! can be approximated by means of the expression $\sqrt{2\pi n}\left(\left(\frac{n}{e}\right)^n\right)$, where e is the base of natural logarithms.

[2] Generalized definition of binomial coefficients: If n is not a positive integer or zero, the binomial expansion of $(1+y)^n$ yields, for -1<y<1, the infinite series $1 + \binom{n}{1}y + \binom{n}{2}y^2 + \binom{n}{3}y^3 + \cdots + \binom{n}{r}y^r + \cdots$
Where $\binom{n}{r} = \frac{n(n-1)\ldots(n-r+1)}{r!}$ for r=1, 2, 3,...